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## **An Endogenous Group Formation Theory of Co-operative Networks**

The Economics of La Lega and Mondragón

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### **Abstract**

This paper develops a theory of endogenous league formation and considers its implications for policy in developing countries. We generalize from features of the two most prominent European co-op leagues, Mondragón and La Lega, to develop the first formal model of the endogenous formation of co-operative networks and their constituent member coops. We show that if co-op leagues are formed through an open membership game, there can be two Nash equilibria, one with and one without a co-op league; and in this case, the equilibrium with a co-op league Pareto dominates the latter. In examining the formation of constituent co-operative firms, we show that, when payoffs to joining a co-op for potential worker members are initially increasing in membership and then decreasing, the outcome of the game depends on the rules of co-op formation. If payoffs are equal to the alternative wage at a single, unique membership size, then open membership and exclusive membership rules of the game yield the same outcome that either no co-op will be formed, or all co-ops formed will have the same number of members; but the coalition unanimity game has a unique outcome with co-op formation. If worker member payoffs exceed the alternative wage, our three alternative rules of co-op formation yield different outcomes. In the open membership game where some workers work for conventional firms, coops will be formed at the largest size for which co-op payoffs are equal to the alternative wage. However, if co-op payoffs exceed the conventional wage only when all workers join coops, then equilibrium co-op sizes can potentially include a wide range of membership sizes. In the exclusive membership game, all co-op sizes in the interval for which co-op payoffs are at least as large as conventional wages are equilibria. .../...

Keywords: co-operatives, networks, game theory, Mondragón, La Lega, Legacoop, labour managed firm

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Finally, in the coalition unanimity game, only co-op sizes at which the highest income per member is achieved are equilibria. Only the latter result corresponds to the traditional neoclassical Ward-Vanek labour managed firm literature (though not necessarily with its comparative statics implications). A series of modelling extensions are discussed. Implications for existing and potential co-op leagues in developing countries are appraised, and implications for policy examined.

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## 1. Introduction

In the Mondragón co-operatives in the Basque country, and La Lega co-ops concentrated in north central Italy, tens of thousands of workers operate competitive self-managed industrial enterprises, which in turn are grouped together in leagues that enable them to reap economies of scale in key services such as R&D, marketing, and finance. These networks are rare—there are fewer than a dozen of them globally—but not unique even in these countries (notably the Valencia network in Spain and three others in Italy). How do such networks of unconventional firms come to exist? Although each has its own history and idiosyncratic features, in this paper we develop a game theoretic model to capture some of the general strategic incentives of individuals to form labour-managed co-operatives, such as those in Italy and Spain, and for these co-operatives to further organize themselves into a league.

We consider a two-stage model. In the first stage, ex-ante identical players make a decision to either work in a conventional firm in return for an exogenously determined payoff or participate in a labour-managed co-operative in return for an (uncertain) share in revenue net of capital costs. We allow multiple symmetrically-sized co-ops to be formed endogenously. In the second stage, the co-operatives can form a league with other co-operatives. The formation and maintenance of such a league is assumed to impose costs on the participating co-ops, to pay for the services provided by the leagues (institutional details are given in the next section). Our objective is to characterize the equilibrium composition of the co-operative league and the size of the conventional sector. Ours is the first paper to apply the recent research on endogenous group formation in a non-co-operative framework to the analysis of co-operatives and labour managed firms (LMFs).

In our model, coalitions emerge endogenously as the Nash equilibria of an announcement game. In the first stage, individual players form labour-managed co-operatives through a well-defined coalition formation game. For example, players may form such co-operatives through an open membership game (Yi and Shin, 2000), an exclusive membership game (Hart and Kurz, 1983) or coalition unanimity game (Bloch, 1995, 1997). Each corresponds to alternative traditions in the literature and institutions. Once the players have made a decision to commit to join one of the co-ops, it is irreversible and they cannot change their decision in the second stage to withdraw from one co-op and join another co-op or work in the conventional sector.<sup>1</sup> For technical reasons, as well as for tractability, we restrict ourselves to symmetric equilibria; therefore, the co-ops from the first stage form the symmetric ‘players’ for the second stage league formation game. We assume that a league is formed according to the dominant coalition open membership game of D’Aspremont et al. (1983). This is a plausible representation of the institutions: organizational constitutions and external legal restrictions act to ensure that co-op leagues do not behave so as to maximize rents of existing members only, and admit new co-op members provided that basic requirements are met. Finally, since the players are assumed to be rational and forward-looking, the two-stage game is solved through standard backward induction techniques.

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<sup>1</sup> This is equivalent to assuming that such moves are prohibitively costly; and indeed one does observe workers spending the rest of their working lives in the coops they join.

In addition to contributing to the literature on LMFs and co-op leagues, the special nature of the problems at hand also lead us to contribute to game theory. In the received literature on the endogenous formation of coalitions, ex-ante symmetric players first form coalitions according to some well-defined rule and then play a non-co-operative game in the second stage contingent on the coalition structure from the first stage. In contrast, in our paper the players play two distinct coalition formation games in the two stages. The two stages are then linked through backward induction: the players look forward to the second stage Nash equilibrium when making their first stage decision. The model is helpful in understanding problems of co-operative formation in developing countries, and in suggesting strategies for selective government assistance in the formation of co-operative leagues.

## **2. Mondragón and La Lega: an overview**

The Mondragón Co-operative Corporation in the Basque region of Spain and La Lega co-operative network in Italy are probably the most striking examples of globally competitive labour managed co-operatives. These networks offer a wide range of specialized services to their member co-ops. This section offers a brief review of some of their key attributes.<sup>2</sup>

### **2.1 La Lega**

La Lega Nazionale delle Co-operative e Mutue (The National League of Co-operative and Mutual Societies, or La Lega) was founded in 1886, and is the oldest and largest co-operative organization in Italy, and among the largest in the world. It is an outgrowth of the Italian labour movement. La Lega includes some 5000 worker co-operatives, a fully autonomous grouping which is the subject of this study, as well as thousands of agricultural consumer co-ops, housing and other specialized co-ops in fields such as fishing and transportation. This autonomy is in sharp contrast to developing countries' typical government organized co-op sectors, which group different types of co-ops under a single ministry dominated by growers and similar co-operative forms rather than labour co-ops, despite their sometimes competing interests. For example, in India, the sugar growers processing co-ops have never considered developing their factories as labour co-ops, because this might compete with the growers' profitability.

The explicit purpose of La Lega has been to promote the development of co-operatives, and the diffusion of co-operative principles in society. La Lega's current mission statement emphasizes three main goals: global competitiveness of member enterprises, the social role of co-ops in solving social problems and improving the general quality of life, and the expansion of workplace democracy. La Lega defines itself as a network of autonomous co-operative enterprises. In addition to individual co-ops as listed above, it includes autonomous regional associations, industrial sector associations, specialized consortia, and a national association that engages in research, lobbying, and other activities of benefit to its members. Participation in La Lega is voluntary; any co-op may secede and become fully independent. The fact that co-ops rarely do so despite required payment of dues and other fees is evidence that La Lega provides valued services to its members. La Lega performs the watchdog role of ensuring that La Lega standards are met by all members. At a national level, the co-operative movement runs different types of specialized services

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<sup>2</sup> For a detailed review see Smith (2001).

through a number of subsidiaries, especially in the fields of finance, training and consulting.

In La Lega, many functions are delegated to consortia and second-level co-operatives.<sup>3</sup> Some thirty specialized institutes and consortia are affiliated with La Lega. Inforcoop coordinates these activities, and also raises funds from public sources for activities such as special training classes, seminars and workshops. SMAER is the in-house organizational development consulting group, which plays an active role in helping the large co-ops maintain a participatory style and co-operative labour-management relations. Comunicazione Italia is the public relations arm of La Lega, which both promotes the image of La Lega as a whole and serves as an advertising agency for co-ops and consortia in the network, and offers an additional arena for the co-ordination of marketing strategies. The SINNEA group focuses on training co-op managers. Editrice Co-operativa is La Lega's publishing group.

Il Consorzio Nazionale Approvvigionamenti (ACAM) is primarily a purchasing consortium, oriented to lowering costs of intermediate goods through negotiation on behalf of consortium members. In addition to receiving the usual declining purchase price with larger orders, ACAM works to achieve additional discounts, in part by negotiating long-term relationships with suppliers who agree to such discounts. Concoop is an industrial consortium that engages in subcontracting across co-ops when large orders are received; it is self-supporting through its 2 percent commission on the value of subcontracted work. ICIE is the innovation and technology transfer group; it is only mentioned here because it is described in greater detail below in the section on innovation. Promosviluppo (which roughly means development promotion) is the second-level co-op charged with starting new co-ops. It conducts feasibility studies on conversions of private firms to La Lega co-ops. It also works extensively in the less developed Mezzogiorno regions, offering co-ops as development strategies in underserved areas. The source of funds for La Lega organizations derives primarily from membership fees, and commissions from consortia participation. Consortia and other groups receive a commission for their services; we list four examples here. The financial arms, notably Fincooper, receive interest payments and other standard intermediation fees; marketing consortia may receive a share of sales value of products marketed; and co-op purchasing agent consortia receive commissions on the savings they engender. In addition, some outside contracts also provide sources of revenue; for example, the consortium ICIE (Institute for Co-operative Innovation) receives funds from public sources to do contract innovation work.

## **2.2 Mondragón**

The precursors to today's Mondragón Co-operative Corporation (MCC) were established through the efforts of an influential parish priest, Don José María. After more than a decade of preparatory work in community organizing and establishing a technical school on democratic principles, under Don José María's guidance in 1956 five engineering graduates of the school founded ULGOR (now known as Fagor), the first industrial co-op in what became the Mondragón system.<sup>4</sup> Don José María suggested the original guidelines of the Mondragón co-operative enterprises, which continue to exist in modified form. The

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<sup>3</sup> This section draws on author interviews.

<sup>4</sup> Author's interview with one of the five, Jesus Larranaga, April 16, 2001.

Mondragón credit union, known as the Caja Laboural Popular (CLP), was established in 1959, which played a crucial role in the rapid development of the Mondragón system. In the decade that followed, dozens of additional industrial co-operatives were developed under the rubric of the CLP; later, most of the co-operatives, and other activities were grouped under what is now known as the Mondragón Corporación Co-operativa (MCC). As David Ellerman (1984) memorably put it, the CLP Entrepreneurial Division functioned in these years as ‘a factory factory’. Today, following extensive consolidation and rationalization, the MCC is comprised of some 75 co-ops organized into financial, industrial, and distribution groups, together with administrative services, marketing, research and training bodies, and foreign subsidiaries, which brings the total number of entities in the group to about 120 (about equal to the maximum number of co-ops prior to the consolidation of the 1990s). Although the co-ops are independent, profit sharing takes place both MCC-wide and at the level of the industrial groupings (such as FAGOR and Donovat).

MCC employed a total workforce of 46,861 as of the end of 1999, with nearly 23,000 co-operative members in 75 core co-ops, and several thousand more probationary members (largely in the fast-expanding Eroski retail group), making it the largest industrial group in the Basque region, and eighth largest in Spain. The MCC contributes about 5 percent of the GDP of the Basque region, excluding indirect contributions of the CLP through housing and its other non-MCC investments. Mondragón co-ops get their investment funds from retained earnings, membership capital fees, loans (and in the case of CLP, return from loans), and some outside contracts.

Although substantial rationalization of product lines has occurred particularly in the 1990s, MCC remains a highly diversified conglomerate. Part of this may be viewed as a strategy to mitigate worker member risks; it may also be understood as stemming from a loan diversification strategy of the CLP itself, which made many of the initial investments in an increasingly diversified group of co-ops in the 1960s. However, as with La Lega, there are substantial interlinkages between MCC co-ops. According to Clamp (2000), ‘The household goods division is treated separately but has close ties in the MCC to the components division. Ties to the automotive industry closely relate machine tools and automotive sectors as well as the components division. The division heads meet with one another on a regular basis to facilitate co-ordination of their activities’. Some other co-ops that were once part of the Mondragón group are no longer part of the modern MCC but still thrive as co-operatives, most prominently the 7-co-op ULMA group (however, this group is currently negotiating to re-enter MCC). In the Mondragón system as a whole (beyond MCC), the CLP financial arm remains very important, although it no longer plays the dominant entrepreneurial and administrative role it played in the earlier years of the group.

The core administrative group MCC has now assumed some of the former explicit and implicit authority of the CLP. It is in charge of conceptualizing overall strategy, and it has at its disposal two significant funds, the Central Interco-operation Fund (FCI), which collects about 10 percent of net earnings for investment in co-ops, temporary subsidies, and the like, and the FEPI education, research, and co-op development fund, with about 2 percent of net earnings. However, its real powers are limited. Its board is controlled by member co-ops; and except in the most egregious of cases, it cannot expel co-ops or replace managers. The historically central Entrepreneurial Group, which was formerly a division of MCC, and before that a division of the CLP, is now an independent co-op

within the network. Lagun-Aro operates the social insurance scheme (including unemployment insurance, the pension system, and health care). Ularco is the co-op group handling legal, administrative and some financial functions, founded in 1965. Ikerlan undertakes research and development functions, founded in 1973; it is a co-op in itself, that now provides services to conventional firms throughout Europe. In addition, Ideko provides R&D for the machine tool grouping, and now gets about 25 percent of its revenue from outside contracts. Lankide is the export group. The consumer durables group FAGOR grew out of Ularco, which was an early (mid 1960s) experiment in extensive inter-co-op co-operation. Also significant is the role of FAGOR as a consolidated brand name, reducing marketing costs, and allowing all to benefit from joint efforts to raise quality—an important area in which co-ops can take advantage of strong complementarities.

The Mondragón system has ten basic principles that have been adhered to since its founding.<sup>5</sup> The first principle is ‘open admission’ which means that membership ‘is open to all men and women who accept the basic principles and can prove themselves professionally capable’ given ‘the practical needs and business requirements’. Second, ‘democratic organization’ meaning that the general assembly is sovereign, and operates on the basis of ‘one member, one vote’. Third, sovereignty of labour, meaning that ‘Labour is granted full sovereignty’ in the co-op organization, and ‘the wealth created is distributed in terms of the labour provided and there is a firm commitment to the creation of new jobs’. Moreover, ‘wealth generated by the co-op ... is distributed among the members in proportion to their labour and not on the basis of their holding in share capital’. Fourth, the ‘instrumental and subordinate nature of capital’ principle, meaning that capital receives only limited remuneration. Fifth, the principle of participatory management. Sixth, the payment solidarity principle, which to some degree limits the pay of managers in relation to that of production workers. Seventh, ‘interco-operation’. meaning co-operation within the MCC network, such as risk pooling across co-ops, and joint research and training. Eighth, social transformation, through support of creation of new co-ops, education and other community development initiatives, and a social security system. Ninth, universality, meaning membership in and support of organizations sharing similar goals. Tenth, the principle of education, support for continuous improvement of skills and knowledge of co-op members and those of the surrounding community. In sum, MCC is held together by a set of shared principles, reinforced with the provision of valued services and other organizational safeguards that are analogous to, but in some ways stronger than, those found in La Lega.

### **2.3 Co-op clusters: general issues**

From these overviews, certain general themes suggest themselves. First, co-op density matters. In addition to Spain and Italy, co-ops are generally found in clusters. Smith (2001) presents arguments that geographic proximity to other co-ops is important for co-op success even when co-ops are not grouped formally into a league, among them:

- a) new managers will more likely have had experience with co-operative management when they take a new managerial job at a co-op;
- b) employees will encounter similarly empowered counterparts in joint ventures, sales, or other market activities, maximizing the benefits of such decentralized authority;

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<sup>5</sup> This section draws extensively from the Mondragón website.

- c) banks will have experience lending to such firms; lending transaction costs are always highest when a bank first lends to a borrower with a very different structure or set of internal or external governing regulations,<sup>6</sup> and similar arguments may apply to insurance and other services;
- d) there are more examples of co-op organizational and other relevant experiments to absorb lessons from in the region or sector; e) in addition to pure spillovers, there will be a ‘thick market’ of specialized suppliers to co-ops, such as consultants in organizational development, improving the probability of a good match;
- e) in the case of involuntary separations, it will be easier for co-ops and workers with relevant co-op experience to find each other, lowering training costs;<sup>7</sup> and
- f) some process innovations may fit with the organizational comparative advantage of co-ops, such as operations utilizing knowledge and skills impacted in the work team, that is, unobservable to management. Such innovations would be selected against when workers have an incentive to shirk; but if co-ops can overcome this problem, through a combination of financial incentives of ownership and the incentive for mutual peer monitoring, they may be efficiently used in co-ops. The more co-ops present, the greater the incentive to invest in such innovations.

Beyond co-op density, an organized league plays several important roles, internalizing some of the externalities, and enabling co-ops to take advantage of economies of scale and scope in provision of such services as finance, R&D, training, organizational development, procurement, and marketing, as well as development of new co-ops, which itself provides benefits to existing ones.

### 3 The basic model

In this paper, we model the formation of co-operatives and leagues as *coalitions*. A *coalition structure* is a partition of the set of players; an element of the partition is called a coalition. Therefore, a player can belong to one, and only one, element of the coalition. We consider a model of endogenous formation of coalitions in order to analyze the strategic incentives of individual players to establish labour-managed co-ops, and these co-ops to further organize themselves into co-operative leagues. We address these issues in the context of non-co-operative games of coalition formation. While at first it may seem ironic to study co-operative formation as a non-co-operative game, we think it is highly appropriate: while successful co-ops, once formed, likely behave internally in ways better modeled as a co-operative game, the problem of establishing co-operatives in the first place presents numerous problems that are better conceptualized as elements of a non-co-operative game.

There is also a technical reason why the tools of co-operative game theory—in particular the characteristic function approach—cannot be used here. The coalition function attaches to each coalition its worth, i.e. what the coalition can achieve irrespective of the behavior of other coalitions. But in our framework the formation of a coalition creates positive

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<sup>6</sup> Interview with Renee Jakobs of the National Cooperative Bank (NCB) Development Corporation, Washington DC, 2001.

<sup>7</sup> Such training costs may include learning how to function effectively as a coop member rather than a hired worker. For an applicable formal search model. Involuntary separations in practice can include worker deaths and company bankruptcies.



externalities for other players; therefore the co-operative approach cannot be applied since the worth of a coalition is influenced by the actions of other coalitions. There has been an attempt to address this through the notion of a partition function but some of the problems persist—see Bloch (1997) for an excellent overview.

The equilibrium coalition structure is a function of the rules according to which players form coalitions. While our models do not capture the full richness of the institutional structures described in the previous section, they do reflect the basic problems of the endogenous formation of co-operatives among potential conventional sector workers, and the further agglomerations of such coalitions into leagues, showing the impact of alternative rules of coalition formations on the resulting equilibria. Thus, our models represent a significant advance over the assumption of exogenous creation of such co-ops and leagues found in the literature to date. We consider interpretations of the alternative models in light of the overview of section 2 throughout the remainder of the paper.

Let  $N$  denote a finite set  $N=\{1,2,\dots,n\}$  of ex-ante identical workers, or players. A representative player from this set is denoted by  $i$ . For simplicity we will assume that each player  $i \in N$  has a perfectly inelastic supply of one unit of labour. Players have two choices about their labour endowment. One possibility before each player is to supply their unit of labour in the conventional profit-maximizing sector in exchange for a fixed exogenously given wage,  $w > 0$ . The second possibility before the player is to pool their labour endowment with those of other players in a labour-managed co-op in exchange for a share of the revenue net of the cost of capital. Let  $n_0$  denote the number of players who have chosen to work for a conventional firm; therefore  $m = n - n_0$  are the number of players who have agreed to form co-operatives.

Consider a labour-managed co-op of size  $h$ . Let  $X$  denote the output produced by the co-op (and  $PX$  the value of output) using labour,  $L=h$ , and the vector of capital inputs,  $\mathbf{K}$  from the production function,  $X = f(h, \mathbf{K})$ . Recall that a co-op with  $h$  members will have  $h$  units of labour by assumption. The capital inputs are purchased in a competitive market at input prices given by the vector  $\mathbf{R}$ . Letting  $x = X/h$  denote the output per co-op member, we can define  $c(x, h)$  as the minimum capital cost per player (potential member) required to produce  $x$ , i.e.:

$$c(x, h) = \min \{ \mathbf{R} \cdot \mathbf{K} / L : f(L, \mathbf{K}) / L = x, L = h \}$$

If the constraint  $L=h$  is binding, and the production shows increasing returns to scale, then the inclusion of new members will imply that  $c(x, h) \geq c(x, h')$  if  $h \leq h'$ .

**Example:** Suppose the production function is Cobb-Douglas and given by  $f = L^\alpha K^\beta$  where  $0 < \alpha, \beta < 1$ . We let  $\alpha + \beta > 1$ , i.e. there are increasing returns to scale in production. It can be verified that:

$$c(x, h) = r x^{\frac{1}{\beta}} \frac{1}{h^\xi}, \quad \xi = \frac{\alpha + \beta - 1}{\beta}$$

Therefore, the cost function of the co-op is decreasing in membership size. In contrast to a worker in the conventional sector, a co-op member may face uncertainty on the demand side and thus risk in net income (we assume away employment risk in the conventional sector). We can capture this by allowing demand  $P(X)$  to be uncertain and represent the

random demand by  $\tilde{P}$ ; therefore, the value of output per co-op member,  $y \equiv Px$ , is uncertain. Further, it is plausible that a larger co-op may face less uncertainty than a smaller one. This can be accommodated by letting the uncertainty be a function of the size of the co-op. Formally, let  $F(h, \tilde{P})$  denote the distribution function of  $\tilde{P}$  faced by a member of a co-op coalition of size  $h$ . Further, being a part of a larger coalition reduces the risk to a player in the sense of either first degree or second degree stochastic dominance. Let  $u$  be any increasing concave function. The payoff to player  $i$  who belongs to a coalition of size  $h$  is given by:

$$\pi(h, \tilde{P}) = \int_0^P [u(y) - c(x, h)] dF(h, \tilde{P})$$

Since the players (members) are ex-ante identical, all players in the same co-op receive the same payoff.<sup>8</sup> This also implies that payoffs do not depend on the identity of the player; all that matters is the size of the co-op to which the player belongs and the level of output. Under our assumptions, payoffs of player  $i$  are increasing in the size of the coalition to which she belongs, i.e. if  $h' > h$  then:

$$\begin{aligned} \pi(h', \tilde{P}) &= \int_0^P [u(y) - c(h', x)] dF(h', \tilde{P}) \\ &\geq \int_0^P [u(y) - c(h, x)] dF(h', \tilde{P}) \\ &\geq \int_0^P [u(y) - c(h, x)] dF(h, \tilde{P}) = \pi(h, \tilde{P}) \end{aligned}$$

where the second inequality follows from the fact that  $u$  is increasing and concave, and  $F$  shows first or second order stochastic dominance.

**Example:** (continued) Consider the Cobb-Douglas production function once again and the certainty case with  $u(Px) = Px$  and  $P$  is constant. The co-op's problem is to maximize value of output per worker net of (minimum) capital costs of production:

$$\text{Max}_x Px - R x^\beta \frac{1}{h^\zeta}$$

Solving for the optimal value of output and substituting in the objective function, the co-ops payoffs are given by:

$$\frac{A}{R^{\beta/(1-\beta)}} P^{1/(1-\beta)} h^\zeta, \quad \zeta = \frac{\alpha + \beta - 1}{1 - \beta}$$

where  $A$  is a constant. Therefore, gross payoff to a co-op is increasing in the size of the co-op. It can be verified that gross payoffs to the co-op are concave if  $\alpha + 2\beta < 2$  and convex otherwise. The gross payoff to any member  $i$  in the co-op is given by:

$$\pi(h) = \frac{A}{R^{\beta/(1-\beta)}} P^{1/(1-\beta)} h^{\zeta-1}$$

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<sup>8</sup> In particular, all members join at the same time, so there are no prior property rights, and no members can be laid off involuntarily.

As described in Section 2 for the cases of Mondragón and La Lega, an important aspect of the formation of co-ops is the (positive) externality, or spillovers, created for other co-ops. We can incorporate spillovers from other coalitions quite easily in this framework. Quite generally, let player  $i$  belong to a coalition of size  $h_q$  and  $0 < \theta < 1$  denote the spillover parameter. Then let:

$$g_q = h_q + \theta \sum_{r \neq q} h_r = (1 - \theta) h_q + (n - n_0) \theta = (1 - \theta) h_q + m \theta$$

We can interpret  $g_q$  as the *effective* coalition size for player  $i$ . Therefore, in the symmetric case, gross payoff to player  $i$  can now be written as a function of own coalition size and  $m$  as (where reference to the parameters  $n$  and  $\Phi$  is suppressed):

$$\pi(h, \tilde{P}, m) = \int_0^p [u(y) - c(g)x] dF(g, \tilde{P}) \quad (1)$$

Each co-op chooses output per member to maximize the (expected) payoffs of the members. This yields the reduced form payoffs  $\pi(h, m)$  for each member of the co-op and a total gross payoff of  $h \pi(h, m)$  to the co-op. The co-ops then organize themselves into a league. Let  $L$  denote the number of co-ops who are members of the league. Since all co-ops are symmetric, with a slight abuse of notation we also let  $L$  denote the set of co-ops in the league. Define the indicator function  $\Phi(L)$  for any co-op  $i$  as  $\Phi(L) = 1$  if  $i \in L$  and  $\Phi(L) = 0$  otherwise. Players who are members of the league benefit from an increase in the size of the league; this direct benefit can be captured generally by a function  $\Phi(L)$ . The literature on co-operatives has noted that non-members also gain from the formation of a league; these indirect spillovers can be captured generally by a function  $\Phi(L, h, m)$ . Therefore, payoff to a co-op  $i$  in the second stage can be written quite generally as:

$$\psi(h \pi(h, m), \chi_i(L) \mu(L) + [1 - \chi_i(L)] \lambda(L, h, m)) \quad (2)$$

In the next section, we consider the endogenous formation of a dominant league.

#### 4 Endogenous formation of leagues

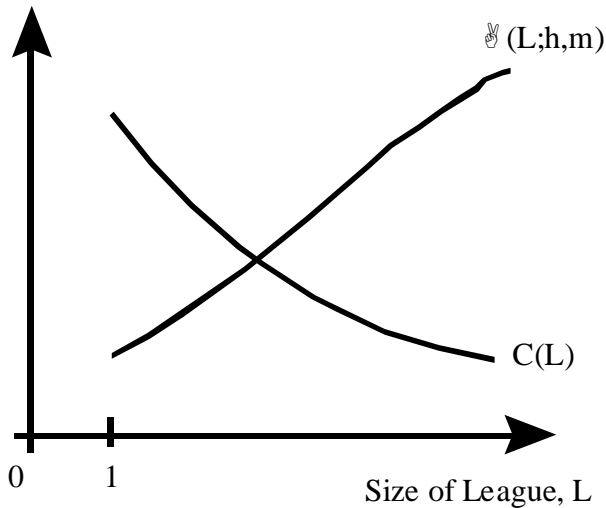
In this section, we consider the incentives of individual co-ops to organize themselves into a league. We take it as given that  $m$  players have formed symmetrically-sized co-ops with  $h$  members in an earlier stage. This allows us to focus exclusively on the endogenous formation of a league. We assume that a dominant coalition of co-ops is formed via an open membership game due to D'Aspremont et al. (1983). In this game, the message space is  $M = \{Y, N\}$ . All co-ops announcing  $Y$  belong to the league while those announcing  $N$  signal their decision not to participate. Membership is open because any player can join the league by announcing  $Y$ . The dominant league is a Nash equilibrium of this game and is internally stable (no co-op who has announced  $Y$  has an incentive to announce  $N$ ) and externally stable (no co-op who has announced  $N$  has any incentive to announce  $Y$ ). In this formulation of a game, only one league is allowed to form; multiple leagues are ruled out by assumption. We do not believe this to be a restriction of our model. One of our objectives is to highlight the potential for *multiple* Nash equilibria in the league formation game, some of which may be inefficient. We are able to show that even when we restrict ourselves to the formation of one dominant league, there are *two* Nash equilibria in the

league formation game, one of which is inefficient. This would hold even if we allowed multiple leagues to form, and indeed in general a larger set of equilibria are likely.

Let  $L$  denote the number of co-ops who have announced  $Y$  and are participating in the league. To normalize for convenient graphical interpretation, we will follow the convention that if  $L=1$ , then there is one co-op in the league (incurring all costs of league formation); if all co-ops choose to be singletons, then  $L=0$ . Consider the gross payoffs to a member in the league. As the arguments considered in section 2 suggest, gross payoffs may depend on the number of players who have agreed to participate in the co-operative sector,  $m$ , and or the number of players in each co-op,  $h$ . In other words, there are spillovers or externalities generated by having more players in the co-operative sector and each co-op in the sector being larger. Consequently, we write gross payoffs quite generally as  $\Phi(L;h,m)$ . It is given by substituting  $\Phi(L)=1$  for a member co-op  $i$  in (2):

$$\Pi(L,h,m) = \psi(h\pi(h,m), \mu(L)) \quad (3)$$

Figure 1: Gross payoffs and costs of a league member as a function of league size



In Figure 2 we plot the *net* payoff to a member of the league. In this figure, we also compare it to the payoff of a non-member. Note first of all that for a non-member, we do not have to distinguish between gross and net payoffs because the non-member does not share any costs of league formation. Second, non-member payoffs will also depend on the tuple  $(h,m)$  because of the presence of spillovers. Third, non-member co-ops can also gain from the presence of a league due to strategic complementarities, as discussed in section 2. Therefore, non-member payoffs are a function of the league size as well and written quite generally as  $\Phi(L;h,m)$ . They are given by substituting  $\Phi(L)=0$  in (2):

$$\Phi(L,h,m) = \psi(h\pi(h,m), \lambda(L,h,m)) \quad (4)$$

These payoffs are shown in Figure 2. Since non-members benefit through positive spillovers from the formation of a league, and the spillover benefits are presumably smaller than direct benefits from participating in a league for small league size, the function  $\Phi$  may intersect  $\Phi$  at least twice. In such a case, it is clear that a co-op would join a league only if its size was in the interval  $(L_1(h,m), L_2(h,m))$ ; otherwise, it would prefer to be a non-

member.<sup>9</sup> After analyzing this case, we will consider some other possibilities for non-member payoffs. In Figure 2, there are two Nash equilibria: no league is formed at all, or a league of size  $L_2(h,m)$  is formed. To see this quite transparently, consider the mapping  $T:I_+ \rightarrow I_+$ , where  $I_+$  is the set of positive integers, such that  $T(L) > L$  if  $\Phi(L;h,m) > \Phi(L;h,m)$  and  $T(L) < L$  if  $\Phi(L;h,m) < \Phi(L;h,m)$ . This mapping is shown in Figure 3.

Figure 2: Net payoffs to member and non-member

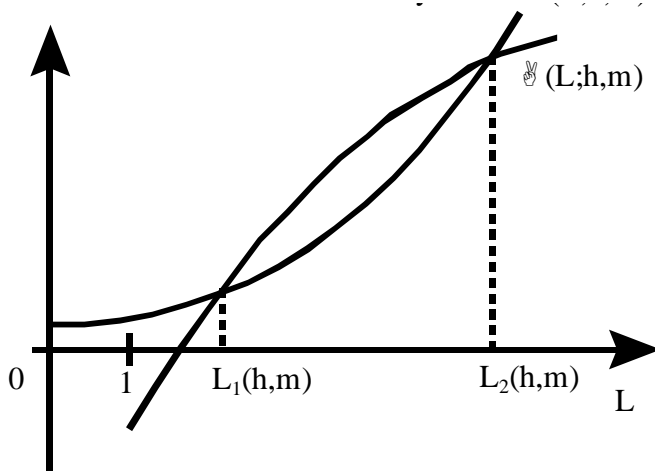
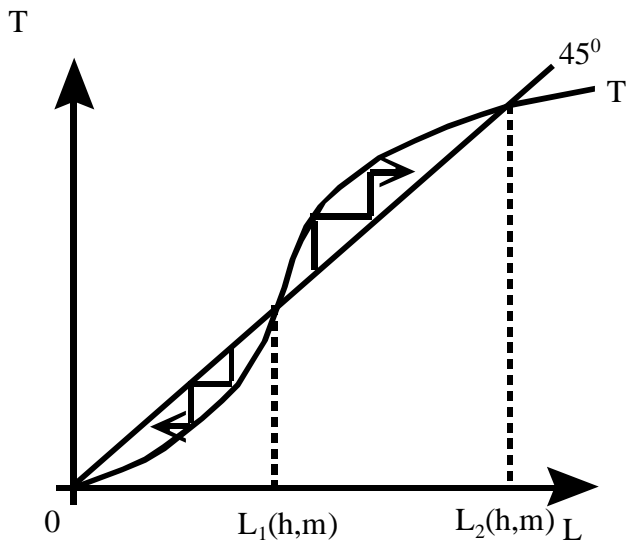


Figure 3: The T Function



It is clear that if the league size is less than  $L_1(h,m)$ , then no co-op would agree to participate in a league. Therefore,  $L=0$  is a Nash equilibrium of the game: if all co-ops are announcing N, then no co-op has any incentive to deviate unilaterally from its message and announce Y instead. If the league size is in the interval  $[L_1(h,m), L_2(h,m))$ , then a non-member co-op has an incentive to change the message from N to Y; likewise, if the league size is strictly greater than  $L_2(h,m)$ , then each member co-op has an incentive to change the

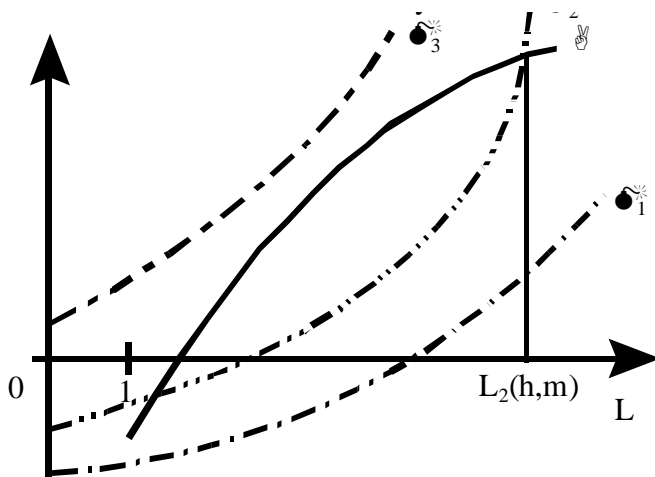
<sup>9</sup> Or in a more general model allowing for multiple leagues, the coop might opt to join an alternative league.

message from Y to N. The league size  $L_2(h,m)$  is the other Nash equilibrium of the game because no member has any incentive to announce N and no non-member has any incentive to announce Y, i.e. the league size is internally and externally stable.

It is interesting to note that with multiple Nash equilibria, payoffs to players in one equilibrium dominate those in another. Only one Nash equilibrium, where a league of size  $L_2(h,m)$  is formed, is efficient. However, our analysis shows that an inefficient equilibrium could arise if all co-ops expect that no league is going to be formed. Even if the efficient league is formed, it is possible given the parameters of the model that all co-ops do not participate in the league. In the case being considered,  $L_2(h,m)$  co-ops are members of the league while  $(m/h)-L_2(h,m)$  co-ops are non-members. Depending on  $(h,m)$  and the shapes of the functions  $\Phi$  and  $\Phi$ , the equilibrium league size could be small and we may have a large number of free-riding non-member co-ops.

There are other possibilities for non-member payoffs as well which are shown in Figure 4. First consider the case where non-member payoffs are given by  $\Phi_1(L;h,m)$ . Since member payoffs dominate non-member payoffs, all co-ops would announce Y and join the league. Therefore, the league would be the grand coalition with all  $m/h$  co-ops as members. In this case, the equilibrium outcome is the efficient outcome as well. Next consider the case where non-member payoffs given by  $\Phi_3(L;h,m)$ . Here, non-member payoffs dominate the member payoffs indicating that spillover effects dominate the direct effects from participating in a league. There is only one Nash equilibrium in this case: no league is formed. All co-ops would like to free ride and announce N. In this case, we see a conflict between the equilibrium outcome and the efficient outcome. In the Nash equilibrium, all co-ops get a payoff equal to  $\Phi_3(0;h,m)$ . But, this is clearly dominated by the member payoffs if the league size is large enough. A non-degenerate league cannot be sustained however. Once any league is formed, each member will have an incentive to defect given that non-member payoffs increase monotonically due to positive spillovers from the formation of a league and dominate member payoffs.

Figure 4: Alternative payoffs to non-members



The last case is where non-member payoffs are given by  $\Phi_2(L;h,m)$ . As in Figure 2, there are 2 possible equilibria, one where no league is formed and the other efficient equilibrium where a league of size  $L_2(h,m)$  is formed. The difference from Figure 2 is that the outcome

of all non-members in the inefficient equilibrium get a negative payoff. In the next section we endogenize the choice of being in a co-operative sector and forming a co-op. Then we will see that if players anticipate that no league will be formed, then they will not form any co-ops either and prefer to work in the conventional sector instead for a fixed wage. We have proved the following general proposition:

**Proposition 1:** *Let  $(h,m)$  be given and assume that a league is formed through the open membership game of D'Aspremont et al. Then, there can be at most two Nash equilibria of the league formation game. If there are two Nash equilibria, then payoffs to co-ops in one Nash equilibrium dominate payoffs in the other.*

**Proof:** We have already argued that there can be at most two Nash equilibria under our assumptions (refer to Figures 3 and 4) at  $L=0$  and  $L_2(h,m) > 0$ . Note that payoffs to the co-ops, whether they belong to the league or not, are equal in the two equilibria, that is:

$$\Pi(0,h,m) = \Phi(0,h,m) , \quad \Pi(L_2(h,m),h,m) = \Phi(L_2(h,m),h,m)$$

From the monotonicity of net payoffs for members and non-members (refer to Figure 2), it follows that:

$$\Pi(0,h,m) \leq \Pi(L_2(h,m),h,m) , \quad \Phi(0,h,m) \leq \Phi(L_2(h,m),h,m)$$

with a strict inequality if the net payoffs for members and non-members is strictly increasing with respect to the size of the league.

## 5. Endogenous formation of co-ops

In the last section, we took the number of players in the co-operative sector as well as the size of each co-op as given in order to focus on the multiple equilibrium problem in the league formation game. In this section we consider the endogenous formation of co-ops. Our objective is to analyze how different rules of coalition formation—in particular, whether membership is exclusive or open—can impact on the equilibrium size of the co-ops.

We now consider the complete two-stage game. In the first stage, ex-ante players decide to either work for a conventional sector for a fixed wage, or join the co-operative sector. If they join the co-operative sector, then they have to decide the size of the co-op. We will restrict ourselves to symmetric equilibria and focus on coalition structures composed of equal-sized co-ops. These symmetric co-ops then become the ‘players’ for the second stage league formation game. In the second stage, given the number of players in the co-operative sector and the size of each co-op, the co-ops play the dominant league formation game discussed in the previous section.

We consider subgame-perfect equilibria. Therefore, using the principle of backward induction, players in the co-operative sector can anticipate the Nash equilibria of the second stage and take these payoffs into account when making their decision in the first stage. The equilibrium outcome in general depends on the rule according to which players form co-ops. We consider three different rules based on how actively the players engage in the process of coalition formation. Since the players are ex-ante symmetric, we only

need characterize the number of coalitions formed in equilibrium and their sizes; the identity of the players in any coalition is irrelevant.

### 5.1 The open membership game

The open membership game was put forward by Yi and Shin (2000). Their construction uses the notion of a message space to co-ordinate the decisions of the players. Let  $\mathbf{M}$  be any arbitrary set with at least  $n$  elements; any  $m \in \mathbf{M}$  is called a message. The players simultaneously announce a message from the set  $\mathbf{M}$ , i.e. the strategy set of each player is  $\mathbf{M}$ . All players who choose the same element  $m$  belong to the same coalition  $C(m)$ . The (Nash) equilibrium coalition structure results when no player has any incentive to unilaterally deviate from its announcement given the announcements of the other players. In the open membership game, the players do not actively maintain the exclusivity (i.e. the size) of the coalition (co-op) to which they belong. Any non-member who announced  $m'$  can potentially join a coalition  $C(m)$  by changing the message from  $m'$  to  $m$ . The open membership structure of this game conforms to an important tradition within the co-operative movement, sometimes associated with Theodor Hertzka's (1891) utopian novel *Freeland*, and reflecting Mondragón's first principle of Open Admission. Although Mondragón's individual co-ops probably do not use purely open membership rules in practice, this game provides a relevant benchmark for analysis.

### 5.2 The exclusive membership game

The version of the exclusive membership game examined here was put forward by Hart and Kurz (1983).<sup>10</sup> In this game, the message space is more specialized and consists of all subsets of the set of players, i.e.  $\mathbf{M} = 2^N - 1$ . All players simultaneously announce coalitions of players to which they wish to belong. Players who have announced the same message  $m$  form a coalition  $C(m)$ .

Consider an example where  $N = \{1, 2, 3, 4, 5\}$ . Players 1, 2 and 3 have announced  $m = \{1, 2, 3, 4\}$  and players 4 and 5 have announced  $m' = \{1, 4, 5\}$ . Then the coalition structure that is induced from these choice of messages is  $\{\{1, 2, 3\}, \{4, 5\}\}$ .

In this game, players have a more active role in determining the coalition structure. The messages that are announced by the players restrict the size of their coalition and prevent outside players from joining. Consider once again the example of the previous paragraph and note that player 5 cannot join the coalition  $\{1, 2, 3\}$  by deviating from  $m'$  to  $m$  unless players in the coalition are willing to include player 5 by changing their announcement to  $m' = \{1, 2, 3, 4, 5\}$  or  $m' = \{1, 2, 3, 5\}$ .

This game reflects the way many co-ops are actually formed in practice. A group of potential members self-selects as a group to form a co-op, without knowing with certainty what the payoff to their coalition will be. As long as the realized payoff, determined after the potential group is formed, at least matches the alternative (conventional firm) wage  $w$ , the outcome is an equilibrium. Formally, recall from Section 3 our distribution function

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<sup>10</sup> To be precise, we utilize the  $\Phi$  version of the exclusive membership game. In the  $\Phi$  version of the game, which we do not use, a coalition is formed if and only if all potential members have announced a coalition comprising of those exact players. In other words, a coalition is formed if and only if there is complete unanimity among the players regarding its composition.



$F(h, \tilde{P})$  reflecting the uncertainty in demand and therefore the value of output. Therefore,  $\Phi(h)$  is the *expected* payoff to each member in the co-op. If the state of nature is such that realized payoffs, given the size of the co-op, are less than  $w$ , then clearly all players will withdraw from the co-op. On the other hand, if realized payoffs is greater than  $w$ , then players have no incentive to withdraw. Therefore, as we will see, the result is a range of equilibrium firm sizes, that may or may not maximize income per member.

### 5.3 The coalition unanimity game

The coalition unanimity game was proposed by Bloch (1995,1997). In this game players move sequentially instead of simultaneously and a coalition is formed if and only if all the potential members of the coalition agree to its formation. The order in which the players move is given exogenously and is common knowledge among the players. Consider for the purpose of illustration the case where  $N=\{1,2,3\}$ . Player 1 moves first and announces a coalition of which it is a member such as  $\{1\}$ ,  $\{1,2\}$ ,  $\{1,3\}$  or  $\{1,2,3\}$ . Suppose player 1 announced  $\{1,2,3\}$ . The other two players now respond in the order 2,3. If both agree, then the coalition is formed. If player 2 disagrees, then 2 proposes a coalition to which it belongs and players 3 and 1 respond sequentially. If player 2 had agreed to the coalition  $\{1,2,3\}$ , but player 3 had disagreed, then 3 gets to announce a coalition to which 1 and 2 respond sequentially. In this game, a coalition structure corresponds to a (subgame perfect) equilibrium if there is no player who wishes to deviate and join another coalition.

Since the players are initially symmetric, the coalition unanimity game is equivalent to the following size announcement game: player 1 announces the size of the coalition of which it is a member. Each prospective member of this coalition responds to the offer by either accepting or rejecting the offer. If all the prospective members accept the offer, the coalition is formed and the procedure is repeated with the excluded players now getting the opportunity to propose the coalition (proceeding sequentially again with the player with the smallest index). If any of the prospective members rejects the offer, then they get an opportunity to propose the size of the coalition they wish to belong to.

The coalition unanimity game requires the most active role by each player in the organization of the coalition. A coalition can only be formed by the unanimous agreement of prospective members. Once players have agreed to participate in a coalition, they are obliged to remain in it and not accept new members at later stages of the game. Further, since the game is sequential (or dynamic), each player looks forward into the game and takes into account the coalition proposals of players who move later. Given that players will therefore be in a position to choose a membership size that maximizes their assumed objective, namely their (equal) share of net income, this game solution concept corresponds to the Ward-Vanek neoclassical analysis of the LMF. However, unanimity might not be reached to lay off members in response to a price increase, under some sets of rules for coalition formation. Note also that the structure of this game also lends itself to an analysis of an 'inegalitarian' co-op of the type introduced by Meade (1972).

Recall from Proposition 1 that there are at most two possible equilibria of the league formation game. Note that in the two second stage equilibria, payoffs to a member and non-member are equalized, i.e.

$$\Pi(0, h, m) = \Phi(0, h, m) , \Pi(L_2(h, m), h, m) = \Phi(L_2(h, m), h, m).$$

We can therefore consider payoffs to a co-op member as a function of the co-op size  $h$  and the number of players in the co-operative sector,  $m$ , without having to differentiate whether the co-op will participate in the league or not. Suppose the co-ops anticipate that they will form a league  $L(h,m)$  where  $L(h,m) = 0$  or  $L(h,m) = L_2(h,m)$ . Then, the payoffs to each member in the co-op will be  $\Phi(h,m) = \Phi(L(h,m);h,m)/h$ . This function can be quite non-linear. It is plausible however that the payoff to each co-op member will increase initially as a function of  $h$  and then decrease (holding  $m$  constant). In addition to plausibility, the basic problems associated with multiple Nash equilibria can be addressed quite transparently with this formulation, as shown in Figure 5.

Figure 5: Payoff for co-op member

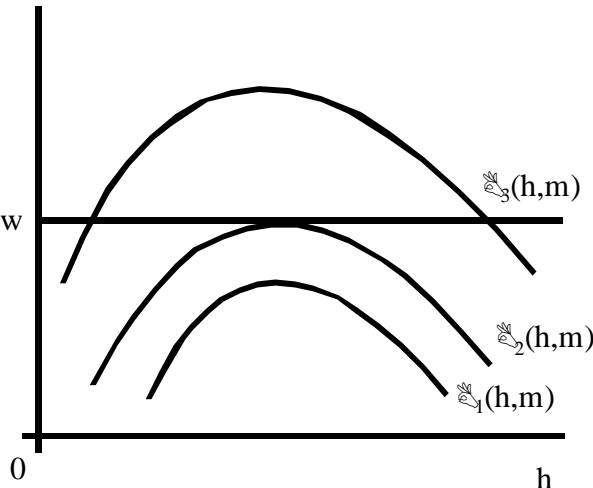
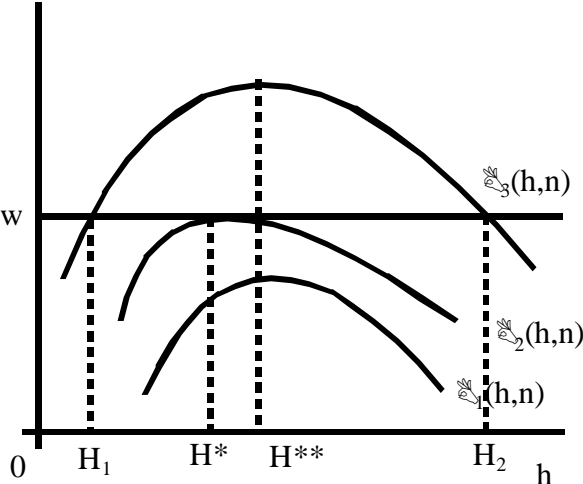


Figure 6: Payoff for co-op member



The number of players in the co-operative,  $m$ , fixes the position of  $\Phi(h,m)$ ; a change in  $m$  will shift the payoff function for the player. In line with the observations in Section II, an increase in  $m$  (and therefore in co-operative density) can be expected to shift the function upwards. Note that the case of  $\Phi_3(h,m)$ , in which payoffs may exceed those of the conventional firm wage, are plausible given the extensive evidence that such co-ops as do

enter the market produce at a higher level of productivity than conventional firms (Bonin, Jones, and Putterman, 1993). We now prove a general result that applies to any second stage Nash equilibrium in the league formation game. We then apply it to the special cases discussed in the previous section. The three cases we consider are shown in Figure 6.

**Proposition 2:** *Consider the payoff function for a member of a co-op that is initially increasing in  $h$  and then decreasing. If:*

(i)  $\pi(h, n) < w$  for all  $h$ , then all three rules of co-op formation yield the same outcome: no co-op will be formed and all players will work in the conventional sector, i.e.  $m=n$ .

(ii)  $\pi(H^*, n) = w$  for a unique value of  $H^*$ , then open membership and exclusive membership yield the same outcomes: either no co-op will be formed or all co-ops formed will have  $H^*$  players. The coalition unanimity game has a unique outcome: only co-ops of size  $H^*$  will be formed.

(iii)  $\pi(h, m) > w$  for some  $h$  and  $m \leq n$ , then the three rules of co-op formation yield different outcomes. In the open membership game, if the stated condition holds for  $m < n$ , then co-ops will be of size  $H_2$ ; on the other hand, if the stated condition holds only for  $m=n$ , then all co-op sizes in the interval  $H^{**} \leq h \leq H_2$  are equilibria. In the exclusive membership game, all co-op sizes in the interval  $H_1 \leq h \leq H_2$  are equilibria. Finally, in the coalition unanimity game, only co-op sizes equal to  $H^{**}$  are equilibria.

**Proof:** (i) If  $\pi(h, n) < w$ , then clearly no player has any incentive to enter the co-operative sector irrespective of the rule of co-op formation.

(ii) This corresponds to the tangency case shown in Figures 5 and 6. In the open membership game, if all players have announced to be singletons, then clearly no player has any incentive to deviate and change his announcement. Similarly, if players have announced a co-op of size  $H^*$ , then players inside the co-op have no incentive to change their message and players outside the co-op have no incentive to change their announcement either. The argument for the exclusive membership game is identical. No player can unilaterally increase the size of the co-op; a player can however unilaterally withdraw from the co-op. But if each co-op is a singleton or composed of  $H^*$  players, then any player will not have any incentive to withdraw from the co-op. Finally consider the coalition unanimity game. The first player to move will announce the size  $H^*$ ; all prospective players in the co-op have no incentive to reject this offer. Since the incentives for player  $H^*+1$  is the same as that for player 1, all co-ops will be of size  $H^*$ .

(iii) Consider the open membership game. If the stated condition holds for  $m < n$ , then payoffs have to be equated in the conventional and co-operative sectors. If the equilibrium co-op size belongs to  $[H_1, H_2)$ , then players in the conventional sector will have an incentive to move to the co-operative sector. Therefore, the only equilibrium size is  $H_2$ . On the other hand, if the stated condition holds only for  $m=n$ , then payoffs in the conventional and co-operative sector do not have to be equalized. Further, the equilibrium size cannot be in the interval  $[H_1, H^*)$  because net payoffs for each player are monotonically increasing over this range; therefore, players in such co-ops will have an incentive to move to another co-op by changing their message. Next consider the exclusive membership game. As in part (ii), players will have no incentive to unilaterally withdraw from a co-op in the range

$[H_1, H_2]$ , nor can they increase the size of the co-op by changing their own announcement. Finally, as in part (ii), under coalition unanimity, the players announcing the co-op size will always have an incentive to announce  $H^{**}$ .

Note that only the coalition unanimity membership game providing the final equilibrium in part iii corresponds to the standard Ward-Vanek neoclassical analysis of the LMF, although even there under some rules of the game outcomes could be closer to those in Steinherr and Thisse (1979), Bonin (1981), and Miyazaki and Neary (1983), that existing memberships would not lay off members among themselves in response to changes in the economic environment, such as a price increase. This lack of correspondence to results not allowing for endogenous membership formation is not necessarily a drawback: the empirical evidence to date has not been supportive of the Ward-Vanek neoclassical analysis (for a survey, see Bonin, Jones, and Putterman, 1993). We can now apply this result to the two extreme subcases discussed in Section 4 where there is a unique Nash equilibrium in the league formation game; the multiple Nash case incorporates both these subcases. Consider first the case where there is a unique second stage equilibrium at  $L=0$ . If  $\Phi(0, h, m) < 0$ , then under all three rules of co-op formation, no co-ops will be formed and all players will work in the conventional sector. What if  $\Phi(0, h, m) > 0$ ? We will then have three possibilities for net payoffs as shown in Figures 5 and 6. In cases (ii) and (iii) discussed in Proposition 2, we will see some players in the co-op sector forming non-singleton co-ops even though they anticipate that no league will be established in the second stage. Further co-op sizes are in general larger under open membership even though net payoffs for each member are maximized under coalition unanimity.

Now consider the case where there is a unique second stage Nash equilibrium at the efficient league level,  $L_2(h, m)$ . In this case, it is clearly efficient for co-ops to form and for co-ops to establish a league. However, it is possible if case (i) in Proposition 2 holds for no co-ops to form at all (even though it is clearly efficient) and for all players to work in the conventional sector. If cases (ii) or (iii) apply, then co-ops will form. Suppose for the sake of argument that a larger co-op size increases the equilibrium league size  $L_2(h, m)$  which in turn increases the net payoffs for the member co-ops. In this situation, it is clear that open membership will strictly dominate coalition unanimity. Under coalition unanimity, the first mover chooses the co-op size that maximizes each member's payoff; however, this may not coincide with the co-op size that maximizes the co-op's payoff in a league. (Of course, the strict domination would work in the other direction if a larger co-op size decreases the equilibrium league size  $L_2(h, m)$ ). Also, open membership weakly dominates exclusive membership (it strictly dominates over the range  $[H_1, H^{**}]$ ; over  $[H^{**}, H_2]$  the outcomes from the two games coincide).

## 6. Conclusions and directions for further research

In this paper, we have generalized from features found in the two most prominent co-op leagues, Mondragón and La Lega, to develop the first formal model of the endogenous formation of co-operative networks and of their constituent member co-ops. We show that if the co-op league is formed through an open membership game, there can be two Nash equilibria. In one equilibrium a co-op league is formed; in the other, it is anticipated that no co-op league will be formed, and hence no co-ops are formed. We show that in this case, the former equilibrium with a co-op league Pareto-dominates the latter, in which no league is formed. In addition, in examining the formation of the constituent individual co-operative firms, we show that, when payoffs to joining a co-op for potential worker

members are initially increasing in the number of members and then decreasing, then the outcome of the game depends on the rules of co-op formation. If the payoffs to individual co-op members are less than the conventional wage, then all three rules of co-op formation yield the outcome that no co-ops will be formed. If the payoffs are exactly equal to the alternative wage at a single, unique membership size, then open membership and exclusive membership rules of the game yield the same outcomes: either no co-op will be formed, or all co-ops formed will have the same number of members. On the other hand, in this case the coalition unanimity game has a unique outcome: only co-ops of that unique size will be formed.

But if payoffs to co-op membership strictly exceed the alternative wage for some membership sizes, then our three alternative rules of co-op formation yield different outcomes. In particular, in the open membership game in the case in which at least some workers continue to work for conventional firms, then co-ops will be formed at the largest size for which co-op payoffs are equal to the alternative wage. However, if co-op payoffs exceed the conventional wage only when all workers join co-ops, then equilibrium co-op sizes can potentially include a wide range of membership sizes. In the exclusive membership game, all co-op sizes in the interval for which co-op payoffs are at least as large as conventional wages are equilibria. Finally, in the coalition unanimity game, only co-op sizes at which the highest income per member is achieved are equilibria. Only the latter result corresponds closely to the traditional neoclassical Ward-Vanek literature (though not necessarily with its comparative statics implications).

In future work we believe that it would be valuable to extend the model to allow for multiple leagues, such as are present in Italy. One way to examine this possibility would be to allow for variable costs of league operation that cause average costs to member co-ops to eventually rise as the number of member co-ops increase. Alternatively, in particular co-op leagues often make strenuous efforts to start new co-ops, an observation not considered in our model. Co-op leagues may have an incentive to do so when efficient league scale has not been reached. Allowing for this phenomenon would add an additional stage to the game. We anticipate that a useful modeling strategy would be to examine co-ops whose upfront organizational costs are high to potential members but lower to an outside entrepreneurial force, such as a co-op league. Alternatively, some potential co-ops might attempt to free ride on the league, to have their organizational costs borne externally. If the government can support the formation of independent leagues, the analysis suggests that this can encourage potential members to form co-ops that otherwise would not emerge. More speculatively, active assistance to such leagues in creating a larger number of constituent co-ops could lead to an improvement in welfare of potential members.

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